ALMOST SUMMING POLYNOMIALS: A COHERENT AND COMPATIBLE APPROACH THE FROM THE INFINITE-DIMENSIONAL HOLOMORPHY VIEWPOINT

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Abstract. In this note we explore the notion of everywhere almost summing polynomials and define a natural norm which makes this class a Banach multi-ideal which is a holomorphy type (in the sense of L. Nachbin) and also coherent and compatible (in the sense of D. Carando, V. Dimant and S. Muro) with the notion of almost summing linear operators. Similar results are not valid for the original concept of almost summing polynomials, due to G. Botelho.

1. Introduction

An operator ideal $\mathcal{I}$ is a subclass of the class $\mathcal{L}$ of all continuous linear operators between Banach spaces such that for all Banach spaces E and F its components

$$\mathcal{I}(E;F) := \mathcal{L}(E;F) \cap \mathcal{I}$$

satisfy:

(a) $\mathcal{I}(E;F)$ is a linear subspace of $\mathcal{L}(E;F)$ which contains the finite rank operators.
(b) If $u \in \mathcal{I}(E;F)$, $v \in \mathcal{L}(G;E)$ and $w \in \mathcal{L}(F;H)$, then $w \circ u \circ v \in \mathcal{I}(G;H)$.

The operator ideal is a normed operator ideal if there is a function $\|\cdot\|_{\mathcal{I}} \colon \mathcal{I} \longrightarrow [0,\infty)$ satisfying

(1) $\|\cdot\|_{\mathcal{I}}$ restricted to $\mathcal{I}(E;F)$ is a norm, for all Banach spaces E, F.
(2) $\|id \colon \mathbb{K} \longrightarrow \mathbb{K} : id(\lambda) = \lambda\|_{\mathcal{I}} = 1$,
(3) If $u \in \mathcal{I}(E;F)$, $v \in \mathcal{L}(G;E)$ and $w \in \mathcal{L}(F;H)$, then $\|w \circ u \circ v\|_{\mathcal{I}} \leq \|w\|\|u\|_{\mathcal{I}}\|v\|$.

When $\mathcal{I}(E;F)$ with the above norm is always complete, $\mathcal{I}$ is called a Banach operator ideal. For details we refer to [20, 34].

The notion of operator ideals, as well as its generalization to the multilinear setting is due to A. Pietsch [34, 35, 36] (we soon will present the precise definitions). A natural question is: given an operator ideal $\mathcal{I}$ (for example the class of compact operators) how to define a multi-ideal and a polynomial ideal that keeps the spirit of $\mathcal{I}$?

Abstract methods of defining when multilinear (and polynomial) extensions of operator ideals are, in some sense, compatible with the structure of the linear ideal were recently discussed by several authors. Sometimes a given operator ideal has several possible extensions to multilinear and polynomial ideals (we mention the case of absolutely summing operators [32, 33] and almost summing operators [5, 29]).

Some methods of evaluating multilinear/polynomial extensions of a given operator ideal have been introduced recently. The idea is that, given positive integers k_1 and k_2, the respective levels of k_1-linearity and k_2-linearity of a given multi-ideal (or polynomial ideal) must have some strong connection and also a connection with the original linear ideal ($k = 1$). In this note we will be interested in the notions of holomorphy types (in the sense of L. Nachbin [27]) and also coherence and compatibility (in the sense of D. Carando, V. Dimant and S. Muro [12, 13, 14]). All these concepts are related to the context of infinite-dimensional holomorphy and for this reason we deal with the case $\mathbb{K} = \mathbb{C}$, but the results can

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be naturally extended to the case of real scalars. For the theory of polynomials in Banach spaces and infinite-dimensional holomorphy we refer to [22, 26].

The main goal of this note is to show that the space of everywhere almost summing multilinear operators can be normed in such a way that it is a Banach multi-ideal and the associated notion of everywhere almost summing polynomials will be a (global) holomorphy type and also coherent and compatible (in the sense of [12]) with the notion of almost summing linear operator. We also show that the previous approach to almost summing polynomials fails these properties.

2. Almost summing operators and polynomials

The idea of considering the notion of nonlinear almost summing operators appears in [4, 5] and was also explored in [28, 29, 32]. For the linear theory of almost summing operators we refer to classical monograph [19]. In this work we aim to explore the concept of almost summing mapping at a given point, obtaining a norm with good properties in the space of almost summing mappings at every point (called everywhere almost summing mappings). Initially we, in part, adapted similar results (for absolutely summing maps) obtained in [3, 7, 25]; however, the case of almost summing mappings is more delicate, and even in the linear case requires special attention (see, [4, Pag. 27]).

Throughout this paper $E, E_1, \ldots, E_n, F, G, G_1, \ldots, G_n, H$ will stand for Banach spaces; B_E denotes the closed unit ball of E and E^* denotes the topological dual of E.

For $1 \leq p < \infty$, the Banach space of all sequences $(x_j)_{j=1}^{\infty}$ in E such that

$$\left\|(x_j)_{j=1}^{\infty}\right\|_p = \left(\sum_{j=1}^{\infty} \|x_j\|^p\right)^{\frac{1}{p}} < \infty$$

is represented by $\ell_p(E)$. We denote by $\ell_p^w(E)$ the linear space formed by the sequences $(x_j)_{j=1}^{\infty}$ in E such that $(\varphi(x_j))_{j=1}^{\infty} \in \ell_p(\mathbb{K})$, for every $\varphi \in E^*$. The space $\ell_p^w(E)$ is a Banach space when endowed with the norm

$$\left\|(x_j)_{j=1}^{\infty}\right\|_{w,p} := \sup_{\varphi \in B_{E^*}} \left(\sum_{j=1}^{\infty} |\varphi(x_j)|^p\right)^{\frac{1}{p}}.$$

The closed linear subspace of $\ell_p^w(E)$ of all sequences $(x_j)_{j=1}^{\infty}$ in $\ell_p^w(E)$, such that

$$\lim_{m\to\infty} \left\|(x_j)_{j=m}^{\infty}\right\|_{w,p} = 0$$

is denoted by $\ell_p^u(E)$.

For $0 < p < \infty$, $Rad_p(E)$ represents the vector space formed by the sequences $(x_j)_{j=1}^{\infty}$ so that

$$\sum_{j=1}^{n} r_j(\cdot)\, x_j$$

converges in $L_p(E) = L_p([0,1]; E)$, where r_j denotes the j^{th} -Rademacher function (or, equivalently, if $\sum_{j=1}^{n} r_j(t)\, x_j$ is convergent in E for (Lebesgue) almost all $t \in [0,1]$). It is well-known (Kahane's Inequality (cf. [19, 11.1])) that $Rad_p(E) = Rad_q(E)$ for all $0 < p, q < \infty$ and for this reason we just write $Rad(E)$ instead of $Rad_p(E)$. The space $Rad(E)$ is a Banach space when endowed with a norm given by

$$\left\|(x_j)_{j=1}^{\infty}\right\|_{Rad(E)} := \left(\int_0^1 \left\|\sum_{j=1}^{\infty} r_j(t)\, x_j\right\|^2 dt\right)^{\frac{1}{2}}.$$

If $1 < p \leq 2$, a continuous linear operator $u : E \to F$ is almost p-summing when $(u(x_j))_{j=1}^{\infty} \in Rad(F)$ whenever $(x_j)_{j=1}^{\infty} \in \ell_p^u(E)$. The theory of almost summing operators has a strong connection with the well-known theory of absolutely summing operators. For details on almost summing operators we refer to the excellent monograph [19] and for recent results we mention [4, 28, 37] and for classical results on absolutely summing linear operators we refer to [18, 19] and references therein; for recent results on linear (and nonlinear) absolutely summing operators we refer to [1, 2, 9, 15, 17, 23, 24, 30, 31] and to [21] for a modern approach to Grothendieck's Resumé and the roots of the theory of absolutely summing operators. The class of almost p-summing operators, denoted by $\Pi_{al,p}$ (with a natural norm that will be clear in the forthcoming Remark 3.8), is Banach operator ideal.

The concept of multi-ideals is also due to A. Pietsch [35]. An ideal of multilinear mappings (or multi-ideal) $\mathcal{M}$ is a subclass of the class $\mathcal{L}$ of all continuous multilinear operators between Banach spaces such that for any positive integer n, Banach spaces $E_1,\dots,E_n$ and F, the components

$$\mathcal{M}(E_1,\dots,E_n;F) := \mathcal{L}(E_1,\dots,E_n;F)\cap\mathcal{M}$$

satisfy:

(a) $\mathcal{M}(E_1,\dots,E_n;F)$ is a linear subspace of $\mathcal{L}(E_1,\dots,E_n;F)$ which contains the n-linear mappings of finite type.

(b) If $T\in\mathcal{M}(E_1,\dots,E_n;F)$, $u_j\in\mathcal{L}(G_j;E_j)$ for $j=1,\dots,n$ and $v\in\mathcal{L}(F;H)$, then $v\circ T\circ(u_1,\dots,u_n)$ belongs to $\mathcal{M}(G_1,\dots,G_n;H)$.

Moreover, $\mathcal{M}$ is a normed multi-ideal if there is a function $\|\cdot\|_{\mathcal{M}}\colon\mathcal{M}\longrightarrow[0,\infty)$ satisfying

(1) For each n, $\|\cdot\|_{\mathcal{M}}$ restricted to $\mathcal{M}(E_1,\dots,E_n;F)$ is a norm, for all Banach spaces $E_1,\dots,E_n$ and F.

(2) $\|A_n\colon\mathbb{K}^n\longrightarrow\mathbb{K} : A_n(\lambda_1,\dots,\lambda_n)=\lambda_1\cdots\lambda_n\|_{\mathcal{M}}=1$ for all n,

(3) If $T\in\mathcal{M}(E_1,\dots,E_n;F)$, $u_j\in\mathcal{L}(G_j;E_j)$ for $j=1,\dots,n$ and $v\in\mathcal{L}(F;H)$, then

$$\|v\circ T\circ(u_1,\dots,u_n)\|_{\mathcal{M}}\le\|v\|\|T\|_{\mathcal{M}}\|u_1\|\cdots\|u_n\|.$$

Analogously, a polynomial ideal is a class $\mathcal{Q}$ of continuous homogeneous polynomials between Banach spaces so that for all $n\in\mathbb{N}$ and all Banach spaces E and F, the components

$$\mathcal{Q}\left({}^nE;F\right):=\mathcal{P}\left({}^nE;F\right)\cap\mathcal{Q}$$

satisfy

(a) $\mathcal{Q}\left({}^nE;F\right)$ is a subspace of $\mathcal{P}\left({}^nE;F\right)$ which contains the finite-type polynomials.

(b) If $u\in\mathcal{L}\left(G;E\right)$, $P\in\mathcal{Q}\left({}^nE;F\right)$ and $w\in\mathcal{L}\left({}^nE;F\right)$, then

$$w\circ P\circ u\in\mathcal{Q}\left({}^nG;H\right).$$

If there exists a map $\|\cdot\|_{\mathcal{Q}}:\mathcal{Q}\to[0,\infty[$ satisfying

(1) For each n, $\|\cdot\|_{\mathcal{Q}}$ restricted to $\mathcal{Q}({}^nE;F)$ is a norm for all Banach spaces E and F;

(2) $\|P_n:\mathbb{K}\to\mathbb{K};\ P_n\left(\lambda\right)=\lambda^n\|_{\mathcal{Q}}=1$ for all n;

(3) If $u\in\mathcal{L}(G;E)$, $P\in\mathcal{Q}({}^nE;F)$ and $w\in\mathcal{L}(F;H)$, then

$$\|w\circ P\circ u\|_{\mathcal{Q}}\le\|w\|\,\|P\|_{\mathcal{Q}}\,\|u\|^n\,,$$

then $\mathcal{Q}$ is a normed polynomial ideal. If all components $\mathcal{Q}\left({}^nE;F\right)$ are complete, $\left(\mathcal{Q},\|\cdot\|_{\mathcal{Q}}\right)$ is called a Banach-ideal of polynomials.

Let $1<p\le2$. Originally, as defined in [4], a polynomial $P\in\mathcal{P}\left({}^nE;F\right)$ is almost summing if $\left(P\left(x_j\right)\right)_{j=1}^{\infty}\in Rad\left(F\right)$ whenever $\left(x_j\right)_{j=1}^{\infty}\in\ell_p^u\left(E\right)$. However, this concept (with any norm) fails to be a global holomorphy type (for the definition of global holomorphy type we refer to [7]). For example, let $P\in\mathcal{P}({}^2c_0;c_0)$ be given by

$$P\left(x\right)=\varphi\left(x\right)x,$$

where $0\neq\varphi\in E^*$, $a\in c_0$ and $\varphi\left(a\right)=1$. From [28, Corollary 3] we have $P\in\mathcal{P}_{al,2}({}^2c_0;c_0)$ but $dP\left(a\right)\notin\mathcal{L}_{al,2}(c_0;c_0)$ and hence $\mathcal{P}_{al,2}$ is not a global holomorphy type. The same example also shows that this concept is not compatible with the ideal of almost summing operators in the sense of [12].

In [28] a new approach to almost summability of arbitrarily nonlinear mappings was presented, where almost summability was considered on certain points of the domain, as we will see in the next section.

3. Everywhere almost summing multilinear mappings and polynomials

Let $1 < p_1, ..., p_n \leq 2$. A map $T \in \mathcal{L}(E_1, \ldots, E_n; F)$ is almost $(p_1, \ldots, p_n)$-summing at $a = (a_1, \ldots, a_n) \in E_1 \times \cdots \times E_n$ if

$$\left(T\left(a_1 + x_j^{(1)}, \ldots, a_n + x_j^{(n)}\right) - T\left(a_1, \ldots, a_n\right)\right)_{j=1}^{\infty} \in Rad\left(F\right)$$

whenever $\left(x_j^{(i)}\right)_{j=1}^{\infty} \in \ell_{p_i}^{u}\left(E_i\right)$, $i = 1, \ldots, n$. The set of all $T \in \mathcal{L}(E_1, \ldots, E_n; F)$ which are almost $(p_1, \ldots, p_n)$-summing at $a = (a_1, \ldots, a_n)$ is denoted by $\mathcal{L}_{al,(p_1,\ldots,p_n)}^{(a)}(E_1, \ldots, E_n; F)$.

In particular, if T is almost $(p_1, \ldots, p_n)$-summing at $a = 0$, we say that T is almost $(p_1, \ldots, p_n)$-summing and we denote the respective set by $\mathcal{L}_{al,(p_1,\ldots,p_n)}(E_1, \ldots, E_n; F)$.

If $E_1 = \ldots = E_n = E$ and $p_1 = ... = p_n = p$ we write $\mathcal{L}_{al,p}^{(a)}(^nE; F)$ or $\mathcal{L}_{al,p}(^nE; F)$ if $a = 0$.

The space composed by all continuous n-linear mappings which are $(p_1, \ldots, p_n)$-summing at every point is denoted by $\mathcal{L}_{al,(p_1,\ldots,p_n)}^{ev}(E_1, \ldots, E_n; F)$. A similar definition holds for polynomials and the respective spaces are represented by $\mathcal{P}_{al,p}^{(a)}(^nE; F)$ and $\mathcal{P}_{al,p}^{ev}(^nE; F)$.

Some of the arguments used in the forthcoming proofs related to multilinear/polynomial notions of almost summing operators can be obtained, *mutatis mutandis*, from similar arguments used for absolutely summing operators in [3, 25]. For this reason we will omit the more simple adaptations and concentrate on the parts that need a different treatment.

3.1. **Dvoretzky-Rogers type theorems.**

The following result was proved in [28, Theorem 4]

Theorem 3.1. *For $1 < p \leq 2$,*

$$\mathcal{P}(^nE; E) \neq \mathcal{P}_{al,p}^{ev}(^nE; E) \Leftrightarrow \dim E = \infty \text{ and}$$
$$\mathcal{L}(^nE; E) \neq \mathcal{L}_{al,p}^{ev}(^nE; E) \Leftrightarrow \dim E = \infty.$$

A repetition of the arguments used in [3, Theorems 3.2 and 3.7] furnishes the following improvements of Theorem 3.1:

Theorem 3.2 (Dvoretzky-Rogers type Theorem for multilinear operators)**.** *Let E be a Banach space, $n \geq 2$ and $1 < p \leq 2$. The following assertions are equivalent:*

(a) E is infinite-dimensional.

(b) $\mathcal{L}_{al,p}^{(a)}(^nE; E) \neq \mathcal{L}(^nE; E)$ for every $a = (a_1, \ldots, a_n) \in E^n$ with either $a_i \neq 0$ for every i or $a_i = 0$ for only one i.

(c) $\mathcal{L}_{al,p}^{(a)}(^nE; E) \neq \mathcal{L}(^nE; E)$ for some $a = (a_1, \ldots, a_n) \in E^n$ with either $a_i \neq 0$ for every i or $a_i = 0$ for only one i.

Theorem 3.3 (Dvoretzky-Rogers type Theorem for polynomials)**.** *Let E be a Banach space, $n \geq 2$ and $1 < p \leq 2$. The following assertions are equivalent:*

(a) E is infinite-dimensional.

(b) $\mathcal{P}_{al,p}^{(a)}(^nE; E) \neq \mathcal{P}(^nE; E)$ for all $a \in E$, $a \neq 0$.

(c) $\mathcal{P}_{al,p}^{(a)}(^nE; E) \neq \mathcal{P}(^nE; E)$ for some $a \in E$, $a \neq 0$.

3.2. **A characterization for the space of everywhere almost summing mappings.**

In order to define an adequate norm on the space $\mathcal{L}_{al,p}^{ev}(E_1, ..., E_n; F)$ we will characterize the maps $T \in \mathcal{L}_{al,p}^{ev}(E_1, ..., E_n; F)$ by means of an inequality (Theorem 3.7).

We will need the Contraction Principle:

Theorem 3.4 ([19, 12.2 - Contraction Principle]). *Let $1 \leq p < \infty$ and consider the randomized sum $\sum_{k=1}^{n} \chi_k x_k$ in the Banach space E. Then, regardless of the choice of real numbers $a_1, \ldots, a_n$,*

$$\left(\int_\Omega \left\|\sum_{k\leq n} a_k \chi_k(\omega) x_k\right\|^p dP(\omega)\right)^{\frac{1}{p}} \leq \left(\max_{k\leq n} |a_k|\right) \cdot \left(\int_\Omega \left\|\sum_{k\leq n} \chi_k(\omega) x_k\right\|^p dP(\omega)\right)^{\frac{1}{p}}.$$

In particular, if A and B are subsets of $\{1, \ldots, n\}$ such that $A \subset B$, then

$$\left(\int_\Omega \left\|\sum_{k\in A} \chi_k(\omega) x_k\right\|^p dP(\omega)\right)^{\frac{1}{p}} \leq \left(\int_\Omega \left\|\sum_{k\in B} \chi_k(\omega) x_k\right\|^p dP(\omega)\right)^{\frac{1}{p}}$$

The following result is a simple consequence of the Contraction Principle:

Lemma 3.5. *If $(x_j)_{j=1}^\infty \in Rad(F)$, then*

$$\|x_n\| \leq 2 \left\|(x_j)_{j=1}^\infty\right\|_{Rad(F)}$$

for all n.

Proof. From the Contraction Principle we know that

$$\left(\int_0^1 \left\|\sum_{j=1}^n r_j(t) x_j\right\|^2 dt\right)_{n=1}^\infty \uparrow \int_0^1 \left\|\sum_{j=1}^\infty r_j(t) x_j\right\|^2 dt.$$

So, for all n, we have

$$\begin{aligned}
\|x_{n+1}\| &= \left(\int_0^1 \left\|\sum_{j=1}^{n+1} r_j(t) x_j - \sum_{j=1}^{n} r_j(t) x_j\right\|^2 dt\right)^{\frac{1}{2}} \\
&\leq \left(\int_0^1 \left\|\sum_{j=1}^{n+1} r_j(t) x_j\right\|^2 dt\right)^{\frac{1}{2}} + \left(\int_0^1 \left\|\sum_{j=1}^{n} r_j(t) x_j\right\|^2 dt\right)^{\frac{1}{2}} \leq 2\left(\int_0^1 \left\|\sum_{j=1}^{\infty} r_j(t) x_j\right\|^2 dt\right)^{\frac{1}{2}}.
\end{aligned}$$

□

From the previous lemma we easily conclude that when $\left(y_j^{(n)}\right)_{j=1}^\infty \in Rad(F)$ and

$$\lim_{n\to\infty} \left(y_j^{(n)}\right)_{j=1}^\infty = (y_j)_{j=1}^\infty \in Rad(F),$$

then

$$\lim_{n\to\infty} y_j^{(n)} = y_j$$

for all j.

The following lemma, which proof can be obtained, *mutatis mutandis*, from [7, Lemma 9.2] is crucial for the proof of the main result of this section:

Lemma 3.6. *If $T \in \mathcal{L}_{al,p}^{ev}(E_1, ..., E_n; F)$ and $(a_1, ..., a_n) \in E_1 \times \cdots \times E_n$, then there is a real number $C_{a_1 \ldots a_n}$ so that*

$$\int_0^1 \left\|\sum_{j=1}^\infty r_j(t) \left(T\left(a_1 + x_j^{(1)}, ..., a_n + x_j^{(n)}\right) - T(a_1, ..., a_n)\right)\right\|^2 dt \leq C_{a_1 \ldots a_n},$$

whenever $\left(x_j^{(k)}\right)_{j=1}^\infty \in B_{\ell_p^u(E_k)}$, $k = 1, ..., n$.

From now on, if $T\in\mathcal{L}_{al,p}^{ev}(E_1,...,E_n;F)$, we consider the n-linear map

$$\phi_T : G_1\times\cdots\times G_n \to Rad(F)$$
$$\phi_T\left(\left(a_1,\left(x_j^{(1)}\right)_{j=1}^{\infty}\right),...,\left(a_n,\left(x_j^{(n)}\right)_{j=1}^{\infty}\right)\right)=\left(T\left(a_1+x_j^{(1)},...,a_n+x_j^{(n)}\right)-T\left(a_1,...,a_n\right)\right)_{j=1}^{\infty},$$

where

$$G_s=E_s\times\ell_p^u\left(E_s\right),\ s=1,..,n,$$

is endowed with the norm

$$\left\|(a,(x_j)_{j=1}^{\infty})\right\|_{G_s}=\max\left\{\|a\|,\left\|(x_j)_{j=1}^{\infty}\right\|_{w,p}\right\}.$$

The next result is inspired on [3] and [25], but some parts (specially the proof of (d)⇒(a)) need a different approach:

Theorem 3.7. *The following assertions are equivalent:*

(a) $T\in\mathcal{L}_{al,p}^{ev}(E_1,...,E_n;F)$.

(b) ϕ_T *is well-defined and continuous.*

(c) There is a $C\geq 0$ *so that*

$$\left(\int_0^1\left\|\sum_{j=1}^{\infty}r_j\left(t\right)\left(T\left(a_1+x_j^{(1)},...,a_n+x_j^{(n)}\right)-T\left(a_1,...,a_n\right)\right)\right\|^2dt\right)^{\frac{1}{2}}\leq C\prod_{k=1}^{n}\left(\|a_k\|+\left\|\left(x_j^{(k)}\right)_{j=1}^{\infty}\right\|_{w,p}\right)$$

for all $\left(x_j^{(k)}\right)_{j=1}^{\infty}\in\ell_p^u(E_k)$, $k=1,...,n$ *and* $(a_1,...,a_n)\in E_1\times\cdots\times E_n$.

(d) There is a $C\geq 0$ *so that*

(3.1)
$$\left(\int_0^1\left\|\sum_{j=1}^{m}r_j\left(t\right)\left(T\left(a_1+x_j^{(1)},...,a_n+x_j^{(n)}\right)-T\left(a_1,...,a_n\right)\right)\right\|^2dt\right)^{\frac{1}{2}}\leq C\prod_{k=1}^{n}\left(\|a_k\|+\left\|\left(x_j^{(k)}\right)_{j=1}^{m}\right\|_{w,p}\right)$$

for all positive integer m, $x_j^{(k)}\in E_k$, $k=1,...,n$ *e* $(a_1,...,a_n)\in E_1\times\cdots\times E_n$.

Proof. *(a)* $\Rightarrow$ *(b)*

Let us show that ϕ_T is continuous. We first show that the set

$$F_{k,\left(x_j^{(1)}\right)_{j=1}^{\infty},...,\left(x_j^{(n)}\right)_{j=1}^{\infty}}=\left\{\begin{array}{c}(a_1,\ldots,a_n)\in E_1\times\ldots\times E_n;\\ \left\|\phi_T\left(\left(a_1,\left(x_j^{(1)}\right)_{j=1}^{\infty}\right),\ldots,\left(a_n,\left(x_j^{(n)}\right)_{j=1}^{\infty}\right)\right)\right\|_{Rad(F)}\leq k\end{array}\right\}$$

is closed for all k and $\left(x_j^{(r)}\right)_{j=1}^{\infty}\in B_{\ell_p^u(E_r)}$. For this task, consider the sets

$$F_{k,\left(x_j^{(1)}\right)_{j=1}^{m},...,\left(x_j^{(n)}\right)_{j=1}^{m}}=\left\{(a_1,\ldots,a_n)\in E_1\times\ldots\times E_n;\left\|\varphi_m(T)\left(a_1,\ldots,a_n\right)\right\|_{Rad(F)}\leq k\right\},$$

where $\varphi_m(T)$ is the map

$$\varphi_m\left(T\right):E_1\times\cdots\times E_n\to Rad(F)$$
$$\varphi_m\left(T\right)\left(a_1,\ldots,a_n\right)=\left(T\left(a_1+x_j^{(1)},\ldots,a_n+x_j^{(n)}\right)-T\left(a_1,\ldots,a_n\right)\right)_{j=1}^{m}.$$

Using the continuity of T we can show that $\varphi_m(T)$ is also continuous. Hence

$$F_{k,\left(x_j^{(1)}\right)_{j=1}^{m},...,\left(x_j^{(n)}\right)_{j=1}^{m}}$$

is closed. Now we will show that

$$F_{k,\left(x_j^{(1)}\right)_{j=1}^{\infty},\dots,\left(x_j^{(n)}\right)_{j=1}^{\infty}} = \bigcap_m F_{k,\left(x_j^{(1)}\right)_{j=1}^{m},\dots,\left(x_j^{(n)}\right)_{j=1}^{m}}.$$

If $(a_1,\dots,a_n) \in F_{k,\left(x_j^{(1)}\right)_{j=1}^{\infty},\dots,\left(x_j^{(n)}\right)_{j=1}^{\infty}}$, then

$$\left(\int_0^1 \left\|\sum_{j=1}^{\infty} r_j(t)\left[T\left(a_1+x_j^{(1)},\dots,a_n+x_j^{(n)}\right)-T(a_1,\dots,a_n)\right]\right\|^2 dt\right)^{1/2} \le k.$$

and using the Contraction Principle we have

$$(a_1,\dots,a_n) \in \bigcap_m F_{k,\left(x_j^{(1)}\right)_{j=1}^{m},\dots,\left(x_j^{(n)}\right)_{j=1}^{m}}.$$

On the other hand, if

$$(a_1,\dots,a_n) \in \bigcap_m F_{k,\left(x_j^{(1)}\right)_{j=1}^{m},\dots,\left(x_j^{(n)}\right)_{j=1}^{m}},$$

then

$$\lim_{m\to\infty}\left(\int_0^1 \left\|\sum_{j=1}^{m} r_j(t)\left[T\left(a_1+x_j^{(1)},\dots,a_n+x_j^{(n)}\right)-T(a_1,\dots,a_n)\right]\right\|^2 dt\right)^{1/2}$$
$$=\left(\int_0^1 \left\|\sum_{j=1}^{\infty} r_j(t)\left[T\left(a_1+x_j^{(1)},\dots,a_n+x_j^{(n)}\right)-T(a_1,\dots,a_n)\right]\right\|^2 dt\right)^{1/2}$$

and since

$$\left(\int_0^1 \left\|\sum_{j=1}^{m} r_j(t)\left[T\left(a_1+x_j^{(1)},\dots,a_n+x_j^{(n)}\right)-T(a_1,\dots,a_n)\right]\right\|^2 dt\right)^{1/2} \le k,$$

for all $m \in \mathbb{N}$, we conclude that

$$(a_1,\dots,a_n) \in F_{k,\left(x_j^{(1)}\right)_{j=1}^{\infty},\dots,\left(x_j^{(n)}\right)_{j=1}^{\infty}}.$$

So the set $F_{k,\left(x_j^{(1)}\right)_{j=1}^{\infty},\dots,\left(x_j^{(n)}\right)_{j=1}^{\infty}}$ is closed.

Now, consider the set

$$F_k := \bigcap F_{k,\left(x_j^{(1)}\right)_{j=1}^{\infty},\dots,\left(x_j^{(n)}\right)_{j=1}^{\infty}},$$

where $\left(x_j^{(r)}\right)_{j=1}^{\infty} \in B_{\ell_p^u(E_r)}, r=1,\dots,n$. From Lemma 3.6 it is obvious that

$$E_1 \times \cdots \times E_n = \bigcup_{k\in\mathbb{N}} F_k.$$

Using Baire Category Theorem we know that there is k_0 so that F_{k_0} has an interior point and repeating the proof of [7, Prop 9.3] (or [3, Theorem 4.1]) we get the result.

$(b) \Rightarrow (c)$ is immediate.

$(c) \Rightarrow (d)$ is also simple.

$(d) \Rightarrow (a)$. We prove the case $n=2$; the other cases are similar. We shall prove that

$$\left(T\left(a_1+x_j^{(1)},a_2+x_j^{(2)}\right)-T(a_1,a_2)\right)_{j=1}^{\infty} \in Rad(F),$$

for all $(a_1,a_2) \in E_1 \times E_2$ and $(x_j^{(i)})_{j=1}^{\infty} \in \ell_p^u(E_i)$, $i=1,2$.

For $a_1 = a_2 = 0$, using (3.1) we can conclude that the sequence

$$S_m(.) = \sum_{j=1}^{m} r_j\,(.)\, T\left(x_j^{(1)}, x_j^{(2)}\right)$$

is a Cauchy sequence on $L_2\,([0,1], F)$, and hence

$$\left(T\left(x_j^{(1)}, x_j^{(2)}\right)\right)_{j=1}^{\infty} \in Rad(F). \tag{3.2}$$

For $a_1 = 0$ and $a_2 \neq 0$, we have

$$\left(T\left(0 + x_j^{(1)}, a_2 + x_j^{(2)}\right) - T(0, a_2)\right)_{j=1}^{\infty} = \left(T\left(x_j^{(1)}, a_2\right) + T\left(x_j^{(1)}, x_j^{(2)}\right)\right)_{j=1}^{\infty}.$$

From our hypothesis, we have

$$\begin{aligned}&\left(\int_0^1 \left\| \sum_{j=1}^{m} r_j\,(t) \left(T\left(0 + x_j^{(1)}, a_2 + x_j^{(2)}\right) - T(0, a_2)\right)\right\|^2 dt\right)^{1/2} \\ &\leq C \left(\left\| \left(x_j^{(1)}\right)_{j=1}^{m} \right\|_{w,p}\right) \left(\|a_2\| + \left\| \left(x_j^{(2)}\right)_{j=1}^{m} \right\|_{w,p}\right)\end{aligned}$$

and hence the sequence $S_m(.) = \sum\limits_{j=1}^{m} r_j\,(.) \left(T\left(x_j^{(1)}, a_2\right) + T\left(x_j^{(1)}, x_j^{(2)}\right)\right)$ is Cauchy on $L_2\,([0,1], F)$; we thus conclude that

$$\left(T\left(x_j^{(1)}, a_2\right) + T\left(x_j^{(1)}, x_j^{(2)}\right)\right)_{j=1}^{\infty} \in Rad(F). \tag{3.3}$$

Now, from (3.2) and (3.3) we get

$$\left(T\left(x_j^{(1)}, a_2\right)\right)_{j=1}^{\infty} \in Rad(F).$$

In a similar way we conclude that

$$\left(T\left(a_1, x_j^{(2)}\right)\right)_{j=1}^{\infty} \in Rad(F)$$

and finally

$$\begin{aligned}&\left(T\left(a_1 + x_j^{(1)}, a_2 + x_j^{(2)}\right) - T\left(a_1, a_2\right)\right)_{j=1}^{\infty} \\ &= \left(T\left(x_j^{(1)}, x_j^{(2)}\right)\right)_{j=1}^{\infty} + \left(T\left(x_j^{(1)}, a_2\right)\right)_{j=1}^{\infty} + \left(T\left(a_1, x_j^{(2)}\right)\right)_{j=1}^{\infty} \in Rad(F).\end{aligned} \tag{3.4}$$

$\square$

Remark 3.8. *When $n = 1$ we recover the concept of almost p-summing linear operators and the infimum of the C satisfying Theorem 3.7 coincides with the norm for the operator ideal $\Pi_{al,p}$.*

4. A well-behaved norm for $\mathcal{L}_{al,p}^{ev}(E_1, ..., E_n; F)$

Using the previous characterization we can consider a natural norm for $\mathcal{L}_{al,p}^{ev}(E_1, ..., E_n; F)$ and show that under this norm $\mathcal{L}_{al,p}^{ev}$ is a Banach multi-ideal. The proofs of Proposition 4.1 and Proposition 4.2 follow the similar results from [3] and we omit:

Proposition 4.1. *The map*

$$\begin{cases} \|\cdot\|_{al,p}^{ev} : \mathcal{L}_{al,p}^{ev}(E_1, ..., E_n; F) \to [0, \infty[\\ \qquad \|T\|_{al,p}^{ev} = \|\phi_T\| \end{cases}$$

is a norm on $\mathcal{L}_{al,p}^{ev}(E_1, ..., E_n; F)$.

Proposition 4.2. *The linear map* $\phi : \mathcal{L}_{al,p}^{ev}(E_1,...,E_n;F) \to \mathcal{L}(G_1,...,G_n;Rad(F))$ *given by* $\phi(T) = \phi_T$ *is injective and its range is closed in* $\mathcal{L}(G_1,...,G_n;Rad(F))$.

So, we easily get the following result:

Proposition 4.3. *The space* $\mathcal{L}_{al,p}^{ev}(E_1,...,E_n;F)$ *is complete under the norm* $\|\cdot\|_{al,p}^{ev}$.

A first step to proving that $\left(\mathcal{L}_{al,p}^{ev}, \|.\|_{al,p}^{ev}\right)$ is a Banach multi-ideal, is to evaluate the norm of $A_n : \mathbb{K}^n \to \mathbb{K}$ given by $A_n(x_1,\ldots,x_n) = x_1 \ldots x_n$. Using that, for every $(a_n)_{n=1}^{\infty} \in \ell_2$,

$$\int_0^1 \left|\sum_{j=1}^{\infty} r_j(t)\, a_j\right|^2 dt = \sum_{j=1}^{\infty} |a_j|^2 ,$$

we can show that $\|A_n\|_{al,p}^{ev} = 1$.

Theorem 4.4. $\left(\mathcal{L}_{al,p}^{ev}, \|.\|_{al,p}^{ev}\right)$ *is a Banach multi-ideal.*

Proof. Let $u_j \in \mathcal{L}(G_j,E_j)$, $j = 1,\ldots,n$, $T \in \mathcal{L}_{al,p}^{ev}(E_1,\ldots,E_n;F)$ and $w \in \mathcal{L}(F;H)$.
Note that

$$\begin{aligned}
&\sum_{j=1}^{m} r_j(t)\left[w\circ T\circ(u_1,\ldots,u_n)\left(a_1+x_j^{(1)},\ldots,a_n+x_j^{(n)}\right) - w\circ T\circ(u_1,\ldots,u_n)(a_1,\ldots,a_n)\right] \\
&= \sum_{j=1}^{m} r_j(t)\left[w\left(T\left(u_1(a_1)+u_1\left(x_j^{(1)}\right),\ldots,u_n(a_n)+u_n\left(x_j^{(n)}\right)\right)\right) - w\left(T\left(u_1(a_1),\ldots,u_n(a_n)\right)\right)\right] \\
&= w\left(\sum_{j=1}^{m} r_j(t)\left[T\left(u_1(a_1)+u_1\left(x_j^{(1)}\right),\ldots,u_n(a_n)+u_n\left(x_j^{(n)}\right)\right) - T\left(u_1(a_1),\ldots,u_n(a_n)\right)\right]\right)
\end{aligned}$$

So we have

$$\begin{aligned}
&\left\|\left(w\circ T\circ(u_1,\ldots,u_n)\left(a_1+x_j^{(1)},\ldots,a_n+x_j^{(n)}\right) - w\circ T\circ(u_1,\ldots,u_n)(a_1,\ldots,a_n)\right)_{j=1}^{m}\right\|_{Rad(H)} \\
&= \left(\int_0^1 \left\|w\left(\sum_{j=1}^{m} r_j(t)\left[\begin{array}{c} T\left(u_1(a_1)+u_1\left(x_j^{(1)}\right),\ldots,u_n(a_n)+u_n\left(x_j^{(n)}\right)\right) \\ -T\left(u_1(a_1),\ldots,u_n(a_n)\right)\end{array}\right]\right)\right\|^2 dt\right)^{\frac{1}{2}} \\
&\leq \left(\|w\|^2 \int_0^1 \left\|\sum_{j=1}^{m} r_j(t)\left[\begin{array}{c} T\left(u_1(a_1)+u_1\left(x_j^{(1)}\right),\ldots,u_n(a_n)+u_n\left(x_j^{(n)}\right)\right) \\ -T\left(u_1(a_1),\ldots,u_n(a_n)\right)\end{array}\right]\right\|^2 dt\right)^{\frac{1}{2}} \\
&\leq \|w\|\,\|T\|_{al,p}^{ev}\left(\|u_1(a_1)\| + \left\|\left(u_1\left(x_j^{(1)}\right)\right)_{j=1}^{m}\right\|_{w,p}\right)\cdots\left(\|u_n(a_n)\| + \left\|\left(u_n\left(x_j^{(n)}\right)\right)_{j=1}^{m}\right\|_{w,p}\right) \\
&\leq \|w\|\,\|T\|_{al,p}^{ev}\,\|u_1\|\cdots\|u_n\|\left(\|a_1\| + \left\|\left(x_j^{(1)}\right)_{j=1}^{m}\right\|_{w,p}\right)\cdots\left(\|a_n\| + \left\|\left(x_j^{(n)}\right)_{j=1}^{m}\right\|_{w,p}\right)
\end{aligned}$$

and it follows that

$$\|w\circ T\circ(u_1,\ldots,u_n)\|_{al,p}^{ev} \leq \|w\|\,\|T\|_{al,p}^{ev}\,\|u_1\|\cdots\|u_n\| .$$

The other properties are easily verified. □

5. The space $\left(\mathcal{P}_{al,p}^{ev}\left({}^{n}E;F\right),\|\cdot\|_{al,p}^{ev}\right)$

In this section we use the following notations:

- If $P\in\mathcal{P}\left({}^{n}E;F\right)$, the unique symmetric n-linear mapping associated to P is denoted by $\overset{\vee}{P}$.
- If $P\in\mathcal{P}\left({}^{n}E;F\right)$, $a\in E$ and $1\leq k\leq n$, then $P_{a^{k}}\in\mathcal{P}\left({}^{n-k}E;F\right)$ is given by
$$P_{a^{k}}(x)=\overset{\vee}{P}(a,...,a,x,...,x).$$
- If $T\in\mathcal{L}\left({}^{n}E;F\right)$ is symmetric, $a\in E$ and $1\leq k\leq n$, then $T_{a^{k}}\in\mathcal{L}\left({}^{n-k}E;F\right)$ is given by
$$T_{a^{k}}(x_{1},...,x_{n-k})=T(a,...,a,x_{1},...,x_{n-k}).$$

By using the Polarization Formula (see [22, Corollary 1.6]) we can easily prove the following result:

Proposition 5.1. *$P\in\mathcal{P}_{al,p}^{ev}\left({}^{n}E;F\right)$ if and only if $\overset{\vee}{P}\in\mathcal{L}_{al,p}^{ev}\left({}^{n}E;F\right)$.*

So, we consider the map
$$\|\cdot\|_{al,p}^{ev}:\mathcal{P}_{al,p}^{ev}\left({}^{n}E;F\right)\rightarrow[0,+\infty)$$
given by
$$\|P\|_{al,p}^{ev}:=\left\|\overset{\vee}{P}\right\|_{al,p}^{ev}.$$
Since $\left(\mathcal{L}_{al,p}^{ev},\|.\|_{al,p}^{ev}\right)$ is a Banach multi-ideal, from [10, Proposition 2.5.2] (see also [7, page 46]) it follows that:

Proposition 5.2. *$\left(\mathcal{P}_{al,p}^{ev},\|\cdot\|_{al,p}^{ev}\right)$ is a (Banach) polynomial ideal.*

We will show that $\left(\mathcal{P}_{al,p}^{ev},\|\cdot\|_{al,p}^{ev}\right)$ is a (global) holomorphy type in the sense of L. Nachbin.

An $(n+1)$-linear map $A\in\mathcal{L}\left({}^{n}E,G;F\right)$ is symmetric in the n *first variables if*
$$A\left(x_{1},\ldots,x_{n},y\right)=A\left(x_{\sigma(1)},\ldots,x_{\sigma(n)},y\right)$$
for all permutation σ in the set $\{1,\ldots,n\}$, for all $x_{1},\ldots,x_{n}\in E$ and $y\in G$.

Below, given $A\in\mathcal{L}\left(E_{1},\ldots,E_{n};F\right)$ and $a\in E_{n}$ we define $Aa\in\mathcal{L}\left(E_{1},\ldots,E_{n-1};F\right)$ by
$$Aa\left(x_{1},\ldots,x_{n-1}\right)=A\left(x_{1},\ldots,x_{n-1},a\right).$$
The following property (Property (B)) was explored in [7]. The name “Property B” is dedicated to Hans-Andreas Braunss, who essentially worked with a similar concept in his PhD thesis [11].

Definition 5.3 (Property (B))**.** *Let $\mathcal{I}$ be a class of continuous multilinear maps between Banach spaces such that for all positive integer n and Banach spaces $E_{1},\ldots,E_{n}$, F the component*
$$\mathcal{I}\left(E_{1},\ldots,E_{n};F\right):=\mathcal{L}\left(E_{1},\ldots,E_{n};F\right)\cap\mathcal{I}$$
is a linear subspace of $\mathcal{L}\left(E_{1},\ldots,E_{n};F\right)$ endowed with a norm represented by $\|\cdot\|_{\mathcal{I}}$. The class $\mathcal{I}$ has the property (B) if there is a $C\geq1$ so that for all $n\in\mathbb{N}$, all Banach spaces E and F and all $A\in\mathcal{I}\left({}^{n}E,\mathbb{K};F\right)$ which is symmetric in the n first variables, it occurs
$$A1\in\mathcal{I}\left({}^{n}E;F\right)\text{ and }\|A1\|_{\mathcal{I}}\leq C\|A\|_{\mathcal{I}}$$

Theorem 5.4. *$\left(\mathcal{P}_{al,p}^{ev},\|.\|_{al,p}^{ev}\right)$ is a global holomorphy type.*

Proof. Standard computations show that $\left(\mathcal{L}_{al,p}^{ev},\|.\|_{al,p}^{ev}\right)$ has property (B) and the result follows from [7, Theorem 3.2]. □

Now we show that, contrary to the original definition from [4], our approach is coherent and compatible with the notion of almost p-summing linear operators.

Definition 5.5 (Carando, Dimant and Muro [12]). *Let $\mathcal{U}$ be a normed ideal of linear operators. A normed ideal of n-homogeneous polynomials $\mathcal{U}_n$ is compatible with $\mathcal{U}$ if there exist positive constants A and B such that for every Banach spaces E and F, the following conditions hold:*

(i) For each $P \in \mathcal{U}_n(E;F)$ and $a \in E$, $P_{a^{n-1}}$ belongs to $\mathcal{U}(E;F)$ and

$$\|P_{a^{n-1}}\|_{\mathcal{U}(E;F)} \leq A\,\|P\|_{\mathcal{U}_n(E;F)}\,\|a\|^{n-1}.$$

(ii) For each $T \in \mathcal{U}(E;F)$ and $\gamma \in E^*$, $\gamma^{n-1}T$ belongs to $\mathcal{U}_n(E;F)$ and

$$\left\|\gamma^{n-1}T\right\|_{\mathcal{U}_n(E;F)} \leq B\,\|\gamma\|^{n-1}\,\|T\|_{\mathcal{U}(E;F)}$$

Definition 5.6 (Carando, Dimant and Muro [12]). *Consider the sequence $\{\mathcal{U}_k\}_{k=1}^{N}$, where for each k, $\mathcal{U}_k$ is a ideal of k-homogeneous polynomials and N is eventually infinite. The sequence $\{\mathcal{U}_k\}_k$ is a coherent sequence of polynomial ideals if there exist positive constants C and D such that for every Banach spaces E and F, the following conditions hold for $k = 1, \ldots, N-1$:*

(i) For each $P \in \mathcal{U}_{k+1}(E;F)$ and $a \in E$, P_a belongs to $\mathcal{U}_k(E;F)$ and

$$\|P_a\|_{\mathcal{U}_k(E;F)} \leq C\,\|P\|_{\mathcal{U}_{k+1}(E;F)}\,\|a\|.$$

(ii) For each $P \in \mathcal{U}_k(E;F)$ and $\gamma \in E^*$, γP belongs to $\mathcal{U}_{k+1}(E;F)$ and

$$\|\gamma P\|_{\mathcal{U}_{k+1}}(E;F) \leq D\,\|\gamma\|\,\|P\|_{\mathcal{U}_k(E;F)}.$$

In our next two results we will represent $\left(\mathcal{P}_{al,p}^{ev}({}^{n}E;F), \|\cdot\|_{al,p}^{ev}\right)$ by $\left(\mathcal{P}_{al,p}^{ev(n)}({}^{n}E;F), \|\cdot\|_{al,p}^{ev(n)}\right)$ in order to be more precise.

Theorem 5.7. *The sequence $\left\{\mathcal{P}_{al,p}^{ev(k)}, \|\cdot\|_{al,p}^{ev(k)}\right\}_{k=1}^{N}$ is coherent.*

Proof. Let $a, b \in E$ and $P \in \mathcal{P}_{al,p}^{ev(k+1)}\left({}^{k+1}E;F\right)$. From the Polarization Formula one can prove that

$$(P_a)^{\vee} = \overset{\vee}{P}_a$$

and from the definition of the norms $\|.\|_{al,p}^{ev(k)}$, a simple calculation shows that

$$\left\|\overset{\vee}{P}_a\right\|_{al,p}^{ev(k)} \leq \left\|\overset{\vee}{P}\right\|_{al,p}^{ev(k+1)}\|a\|.$$

So we obtain

$$\|P_a\|_{al,p}^{ev(k)} = \left\|(P_a)^{\vee}\right\|_{al,p}^{ev(k)} = \left\|\overset{\vee}{P}_a\right\|_{al,p}^{ev(k)} \leq \left\|\overset{\vee}{P}\right\|_{al,p}^{ev(k+1)}\|a\| = \|P\|_{al,p}^{ev(k+1)}\|a\|$$

and (i) of Definition 5.6 is satisfied.

Now, consider $\gamma \in E^*$ and $P \in \mathcal{P}_{al,p}^{ev(k)}\left({}^{k}E;F\right)$. By invoking the Polarization Formula we have

$$(\gamma P)^{\vee} = \gamma\overset{\vee}{P}$$

and from the definition of the norms $\|.\|_{al,p}^{ev(k)}$ it is not very difficult to prove that

$$\left\|\gamma\overset{\vee}{P}\right\|_{al,p}^{ev(k+1)} \leq \|\gamma\|\left\|\overset{\vee}{P}\right\|_{al,p}^{ev(k)}.$$

Hence

$$\|\gamma P\|_{al,p}^{ev(k+1)} = \left\|(\gamma P)^{\vee}\right\|_{al,p}^{ev(k+1)} = \left\|\gamma\overset{\vee}{P}\right\|_{al,p}^{ev(k+1)} \leq \|\gamma\|\left\|\overset{\vee}{P}\right\|_{al,p}^{ev(k)} = \|\gamma\|\,\|P\|_{al,p}^{ev(k)}$$

and the result follows. □

Theorem 5.8. *For every positive integer k, the polynomial ideal* $\left(\mathcal{P}_{al,p}^{ev(k)}, \|\cdot\|_{al,p}^{ev(k)}\right)$ *is compatible with* $\Pi_{al,p}$.

Proof. Let $u \in \Pi_{al,p}(E;F)$ and $\gamma \in E^*$. From the proof of the Proposition 5.7 it follows that

$$\gamma u \in \mathcal{P}_{al,p}^{ev(2)}\left({}^2E;F\right)$$

and

$$\|\gamma u\|_{al,p}^{ev(2)} \leq \|\gamma\| \, \|u\|_{al,p}$$

Analogously,

$$\gamma^2 u \in \mathcal{P}_{al,p}^{ev(3)}\left({}^3E;F\right)$$

and

$$\left\|\gamma^2 u\right\|_{al,p}^{ev(3)} \leq \|\gamma\| \, \|\gamma u\|_{al,p}^{ev(2)} \leq \|\gamma\|^2 \, \|u\|_{al}.$$

Proceeding by induction, we have (ii) of Definition 5.5.

Now, let $a \in E$ and $P \in \mathcal{P}_{al,p}^{ev(n)}\left({}^nE;F\right)$. Then, from the proof of the Proposition 5.7 we have

$$P_a \in \mathcal{P}_{al,p}^{ev(n-1)}\left({}^{n-1}E;F\right)$$

and

$$\|P_a\|_{al,p}^{ev(n-1)} \leq \|P\|_{al,p}^{ev(n)} \, \|a\|.$$

The proof follows by induction. □

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